Singularity-softening prescription for the Bethe-Salpeter equation.

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The reduction of the two fermion Bethe-Salpeter equation in the framework of light-front dynamics is studied for one gauge $A^+ = 0$. The arising effective interaction can be perturbatively expanded in powers of the coupling constant g, allowing a defined number of gauge boson exchanges. The singularity of the kernel of the integral equation at vanishs plus momentum of the gauge is canceled exactly in on approuch. We studied the problem using a singularity-softening prescription for the light-front gauge.

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I. INTRODUCTION

The Bethe-Salpeter equation for ground state of two fermions exchanging gauge boson presents divergences in the momentum q^+, even in the ladder aproximation project in the light front [1]. Gauge theories with light front gauge, also present the difficulty associated to the instantaneous term of the propagator of a system composed by fermions boson-exchanger interaction. We used a prescription that allowed an apropriate description of the singularity in the propagator of the gauge boson in the light front.

We start by considering in Section II gauge boson propagator is canonical procedure of determining the propagator and projected in front of light, that is, for intervals of time x^+. In section III we obtain the two free propagator only propagating modes are the physical. In Section IV we calculated the correction to the propagator of two fermions particles spreading for the future in the light-front time change a gauge boson g^2. In Section V we used the hierarchical equations to obtain Bethe-Salpeter equation. In Section VI we presented the in the light front Bethe-Salpeter equation. In Section VII we presented our conclusions.

II. GAUGE BOSON PROPAGATOR

The Lagrangian density for the vector gauge field (for simplicity we consider an Abelian case) is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}\left(n_\mu A^\mu\right)^2 - \frac{1}{2\beta}\left(\partial_\mu A^\mu\right)^2 , \tag{1}$$

where α and β are arbitrary constants[2]. Of course, with these additional gauge breaking terms, the Lagrangian density is no longer gauge invariant and as such gauge fixing problem in this sense do not exist anymore. Now, $\partial_\mu A^\mu$ doesn't need to be zero so that the Lorentz condition is verified [3].

Here, instead of going through the canonical procedure of determining the propagator as done in the previous section, we shall adopt a more head-on, classical procedure by looking for the inverse operator corresponding to the differential operator sandwiched between the vector potentials in the Lagrangian density. For the Abelian gauge field Lagrangian density we have:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\beta}\left(\partial_\mu A^\mu\right)^2 - \frac{1}{2\alpha}\left(n_\mu A^\mu\right)^2 = \mathcal{L}_{\rm E} + \mathcal{L}_{GF} \tag{2}$$

By partial integration and considering that terms which bear a total derivative don't contribute and that surface terms vanish since $\lim\limits_{x\to\infty} A^\mu(x) = 0$, we have

$$\mathcal{L} = \frac{1}{2}A^\mu \left(\Box g_{\mu\nu} - \partial_\mu\partial_\nu + \frac{1}{\beta}\partial_\mu\partial_\nu - \frac{1}{\alpha}n_\mu n_\nu\right) A^\nu \tag{3}$$

To find the gauge field propagator we need to find the inverse of the operator between parenthesis in (3). That differential operator in momentum space is given by:

$$O_{\mu\nu} = -k^2 g_{\mu\nu} + k_\mu k_\nu - \theta k_\mu k_\nu - \lambda n_\mu n_\nu\, , \tag{4}$$

where $\theta = \beta^{-1}$ and $\lambda = \alpha^{-1}$, so that the propagator of the field, which we call $S^{\mu\nu}(k)$, must satisfy the following equation:

$$O_{\mu\nu} S^{\nu\lambda}(k) = \delta_\mu^\lambda \tag{5}$$

$S^{\nu\lambda}(k)$ can now be constructed from the most general tensor structure that can be defined, i.e., all the possible linear combinations of the tensor elements that composes it [4]:

$$\begin{aligned} S^{\mu\nu}(k) &= g^{\mu\nu}A + k^\mu k^\nu B + k^\mu n^\nu C + n^\mu k^\nu D + k^\mu m^\nu E + \\ &\quad + m^\mu k^\nu F + n^\mu n^\nu G + m^\mu m^\nu H + n^\mu m^\nu I + m^\mu n^\nu J \end{aligned} \tag{6}$$

where m^μ is the light-like vector dual $(m^\mu = n^{*\mu} \equiv (n^0, -\overrightarrow{n}))$ to the n^μ, and A, B, C, D, E, F, G, H, I and J are coefficients that must be determined in such a way as to satisfy (5). Of course, it is immediately clear that since (3) does not contain any external light-like vector m_μ, the coefficients $E = F = H = I = J = 0$ straightaway. We have

$$\begin{aligned} S^{\mu\nu}(q) &= -\frac{1}{q^2}\left\{ g^{\mu\nu} + \frac{\left(\alpha q^2 + n^2\right)(\beta - 1)}{\left(\alpha q^2 + n^2\right) q^2 + q^{+2}(\beta - 1)} q^\mu q^\nu - \frac{q^+ (\beta - 1)\left(q^\mu n^\nu + n^\mu q^\nu\right)}{\left(\alpha q^2 + n^2\right) q^2 + q^{+2}(\beta - 1)} + \right. \\ &\quad \left. - \left[\frac{1}{\left(\alpha q^2 + n^2\right) q^2 + q^{+2}(\beta - 1)}\right] n^\mu n^\nu q^2 \right\} \end{aligned}$$

Again, it is a matter of straightforward algebraic manipulation to get the relevant propagator in the light-front gauge $n^2 = 0$ and taking the limit $\alpha,\beta \to 0$, we have

$$S^{\mu\nu}(k) = -\frac{1}{q^2}\left\{ g^{\mu\nu} - \frac{q^\mu n^\nu + n^\mu q^\nu}{q^+} + \frac{n^\mu n^\nu}{q^{+2}} q^2 \right\}, \tag{7}$$

which has the outstanding third term commonly referred to as *contact term*. Now we needed a prescription to treat the singularity in q^+.

III. ZERO MODES THROUGH THE SINGULARITY-SOFTENING PRESCRIPTION

Zero modes in the light front milieu is a very subtle problem which for years have been challenging us with the best of our efforts to understand it, to make it manageable and to make some physical sense out of it. We have already learned that a prescription to handle those singularities cannot be solely mathematically well-defined — that is not enough — we now know that the prescription must be causal, that is, it needs to ascertain that its implementation does not violate causality [5]. ML prescription has been heralded as the causal prescription to handle the light-front singularites and many a calculation do confirm that it has solved many difficulties concerning one- and two-loop quantum corrections to Feynman diagrams. However, as seen in the previous sections, ML prescription does not remove the pathological zero modes in the one- and two-vector gauge boson propagation at the quantum level. We therefore come to the place most important in this work: The introduction of a novel prescription that is causal and can handle the light-front singularities which does not leave remnant zero modes is presented and applied to the one and two propagating vector bosons in the light-front gauge.

The index SS stands for this singularity-softening prescription for the treatment of the $(k^+)^{-1}$ poles (cf.[6]), namely,

$$\begin{aligned} \left[\frac{1}{k^+}\right]_{\mathrm{SS}} &= \lim_{\varepsilon\to 0}\left[\frac{k^2}{k^+\left(k^2 + i\varepsilon\right)}\right]_{\mathrm{SS}} \\ &= \lim_{\varepsilon\to 0}\left[\frac{k^- - k^-_{\mathrm{on}}}{k^+\left(k^- - k^-_{\mathrm{on}} + \frac{i\varepsilon}{k^+}\right)}\right]_{\mathrm{SS}} \end{aligned} \tag{8}$$

We use for one gauge boson. Like this, we will make a case for to component $S^{\perp -}$ is:

$$\begin{aligned} S^{(1)\perp -}(p^-) &= i\int dk_1^- \frac{k_1^\perp\, n^- \delta\left(p^- - k_1^-\right)}{k_1^+\left(k_1^- - \frac{k_1^{\perp 2}}{k_1^+} + \frac{i\varepsilon}{k_1^+}\right)}\left[\frac{1}{k_1^+}\right]_{\mathrm{SS}} \\ &= i\int dk_1^- \frac{k_1^\perp\, n^- \delta\left(p^- - k_1^-\right)}{k_1^+\left(k_1^- - \frac{k_1^{\perp 2}}{k_1^+} + \frac{i\varepsilon}{k_1^+}\right)}\left[\frac{k_1^- - k^-_{1\mathrm{on}}}{k_1^+\left(k_1^- - k^-_{1\mathrm{on}} + \frac{i\varepsilon}{k_1^+}\right)}\right]_{\mathrm{SS}} \end{aligned} \tag{9}$$

The result is:

$$S^{(1)\perp -}(p^-) = \frac{\theta(p^+)\, p^\perp n^-}{p^+}\left[\frac{p^- - p^-_{\rm on}}{p^+\left(p^- - p^-_{\rm on} + \frac{i\varepsilon}{p^+}\right)}\right]_{\rm SS}\frac{i}{\left(p^- - K_0^{(1)-} + i\varepsilon\right)}, \tag{10}$$

where we have introduced the definition

$$K_0^{(1)-} = p^-_{\rm on} = \frac{p^{\perp 2}}{p^+}, \tag{11}$$

However, since the Dirac delta funtion $\delta\left(p^- - k_1^-\right)$ forces us onto the mass-shell, the numerator is identically zero, that is,

$$p^- - p^-_{\rm on} = 0$$

and this is true for massless as well as massive gauge bosons.

Thus, finally

$$S^{(1)\perp -}(p^-) = 0.$$

For the component S^{--} we have $S^{(1)--}(p^-) = 0$.

In the case of $S^{(1)\perp\perp}(p^-)$ component we have

$$S^{(1)\perp\perp}(P^-) = \frac{\theta(p^+)}{p^+}\frac{i\left(-g^{\perp\perp}\right)}{\left(p^- - K_0^{(1)-} + i\varepsilon\right)}$$

Clearly, this case does not present us with the $p^+ = 0$ difficulty, and the only non-vanishing result is just $S^{(1)\perp\perp}$. Only the physical degrees of freedom (transverse ones) do propagate and without zero mode hindrances anywhere!

The two gauge bosons case, $S^{(2)\perp -,\perp -}$ we have

$$\begin{aligned} S^{(2)\perp -,\perp -}(P^-) &= -\frac{1}{(2\pi)}\int \frac{dk_1^-}{k_1^+\left(P^+ - k_1^+\right)}\left[\frac{k_1^- - k^-_{1\rm on}}{k_1^+\left(k_1^- - k^-_{1\rm on} + \frac{i\varepsilon}{k_1^+}\right)}\right]_{\rm SS}\frac{n^- k_1^\perp}{\left(k_1^- - \frac{k_1^{\perp 2}}{k_1^+} + \frac{i\varepsilon}{k_1^+}\right)} \\ &\times \left[\frac{P^- - k_1^- - k^-_{2\rm on}}{\left(P^+ - k_1^+\right)\left(P^- - k_1^- - k^-_{2\rm on} + \frac{i\varepsilon}{P^+ - k_1^+}\right)}\right]_{\rm SS}\frac{n^- k_2^\perp}{\left(P^- - k_1^- - \frac{\left(P^\perp - k_1^\perp\right)^2}{P^+ - k_1^+} + \frac{i\varepsilon}{P^+ - k_1^+}\right)}\ . \end{aligned}$$

Evaluating the residue at the pole

$$k_1^- = k^-_{1\rm on} - \frac{i\varepsilon}{k_1^+}\ , \tag{12}$$

we have

$$S^{(2)\perp -,\perp -}(P^-) = 0$$

For the other component we have

$$\begin{array}{ll} S^{(2)--,--}(P^-) = 0 & S^{(2)\perp -,--}(P^-) = 0 \\ S^{(2)\perp -,\perp\perp}(P^-) = 0 & S^{(2)--,\perp\perp}(P^-) = 0 \end{array}$$

Finally, for the component $S^{(2)\perp\perp,\perp\perp}$ result

$$S^{(2)\perp\perp,\perp\perp}(P^-) = -\frac{1}{(2\pi)}\int \frac{dk_1^-}{k_1^+\left(P^+ - k_1^+\right)}\frac{\left(-g^{\perp\perp}\right)}{\left(k_1^- - \frac{k_1^{\perp 2}}{k_1^+} + \frac{i\varepsilon}{k_1^+}\right)}\frac{\left(-g^{\perp\perp}\right)}{\left(P^- - k_1^- - \frac{\left(P^\perp - k_1^\perp\right)^2}{P^+ - k_1^+} + \frac{i\varepsilon}{P^+ - k_1^+}\right)}\ .$$

which yields the non-vanishing contribution

$$S^{(2)\perp\perp,\perp\perp}(P^-) = \frac{\theta(k_1^+)\theta(P^+ - k_1^+)}{k_1^+\left(P^+ - k_1^+\right)} \frac{i\left(-g^{\perp\perp}\right)\left(-g^{\perp\perp}\right)}{\left(P^- - K_0^{(2)-} + i\varepsilon\right)},$$

which is the same as that obtained through the ML-prescription [7].

Therefore, as long as we treat the troublesome zero modes $k^+ = 0$ via the singularity-softening prescription (8) the only non-vanishing component of the two gauge boson propagator is the $(\perp\perp, \perp\perp)$-component, so that there is no zero mode problem left and the only propagating modes are the physical, transverse ones!

IV. GREEN'S FUNCTION $\mathcal{O}(g^2)$

To proceed we calculated the correction to the propagator of two particles fermionic spreading for the future in the time in front of light changing a gauge boson, in which the density Lagrangeana is given for

$$\mathcal{L}_I = g\overline{\Psi}_1\gamma_\mu A^\mu \Psi_1 + g\overline{\Psi}_2\gamma_\nu A^\nu \Psi_2. \tag{13}$$

The fields Ψ_1 and Ψ_2 they correspond to the fermions with mass m_1 and m_2, that we will assume same $m_1 = m_2 = m$, and the field A^μ it corresponds to the boson of intermediate gauge of null mass. The coupling constant is g.

The correction to the propagator of two fermions corresponding to a diagram of the type ladder, it is made calculations, in order g^2. For that we will make the use just of the term propagate of the propagators fermionic of the external lines.

The perturbative correction to the two-body propagator which comes from the exchange of one intermediate virtual boson, is given by

$$\Delta S_{g^2}(x^+) = -(ig)^2 \int d\overline{x}_1^+ d\overline{x}_2^+ S_{k'}(x^+ - \overline{x}_1^+)\gamma_\mu^{(1)} S_k(\overline{x}_1^+) S_q^{\mu\nu}(\overline{x}_1^+ - \overline{x}_2^+) S_{p'}(x^+ - \overline{x}_2^+)\gamma_\nu^{(2)} S_p(\overline{x}_2^+). \tag{14}$$

The intermediate gauge boson, q, propagates between the time interval $\overline{x}_1^+ - \overline{x}_2^+$. The labels in the particle propagators p and k indicates initial and p' and k' final states. Propagator in the light-front gauge is

$$S_q^{\mu\nu}(\overline{x}_1^+ - \overline{x}_2^+) = \frac{1}{2}\int \frac{dq^-}{2\pi} \frac{iN^{\mu\nu}(q) e^{-\frac{i}{2}(\overline{x}_1^+ - \overline{x}_2^+)}}{(q^+)^3\left(q^- - \frac{q^{\perp 2}}{q^+} + \frac{i\varepsilon}{q^+}\right)}, \tag{15}$$

where

$$N^{\mu\nu} = -q^{+2}g^{\mu\nu} + q^+(q^\mu n^\nu + n^\mu q^\nu) - n^\mu n^\nu q^2, \tag{16}$$

$q^+ = k'^+ - k^+$ and $n^\mu = (0, 2, 0^\perp)$.

This derivation is quite straightforward. However, the resulting equation does not make sense as it stands. The most pressing problem is that there are infrared divergences of the form $\int \frac{dq^+}{q^+} = \ln(\infty)$.

The 1-body Green's functions can be derived from the covariant propagator for 1-particles propagating at equal light-front times. In this case the propagator from $x^+ = 0$ to $x^+ > 0$ is given by

$$S(x^+) = \frac{1}{2}\int \frac{dk^- dk^+ dk^\perp}{(2\pi)} \frac{ie^{\frac{-i}{2}k^- x^+}}{k^+\left(k^- - \frac{k_\perp^2 + m^2}{k^+} + \frac{i\varepsilon}{k^+}\right)}. \tag{17}$$

The Fourier transform to the total light-front energy (P^-) is given by $S(P^-) = \frac{1}{2}\int dx^+ e^{\frac{i}{2}P^- x^+} S(x^+)$ and the free 1-body Green's function is given by $S(k^-) = \frac{1}{k^+}G(k^-)$, where

$$G_0^{(1)}(k^-) = \frac{\theta(k^+)}{k^- - k_{on}^-} \tag{18}$$

with $k_{on}^- = \frac{k_\perp^2 + m^2}{k^+}$ being the light-front Hamiltonian of the free 1-particle system.

Let $S_{\rm F}$ denote fermion field propagator in covariant theory

$$S_{\rm F}(x^\mu) = \int \frac{d^4k}{(2\pi)^4}\frac{i(\not{k}_{\rm on}+m)}{k^2-m^2+i\varepsilon}e^{-ik^\mu x_\mu}, \tag{19}$$

where $\not{k}_{\rm on} = \frac{1}{2}\gamma^+\frac{(k^\perp)^2+m^2}{k^{+2}} + \frac{1}{2}\gamma^- k^+ - \gamma^\perp k^\perp$. Using light-front variables in the Eq.(19), we have

$$S_{\rm F}(x^+) = \frac{i}{2}\int\frac{dk^- dk^+ dk^\perp}{(2\pi)}\left[\frac{\not{k}_{on}+m}{k^+\left(k^- - k^-_{on}+\frac{i\varepsilon}{k^+}\right)} + \frac{\gamma^+}{2k^+}\right]e^{\frac{-i}{2}k^- x^+}. \tag{20}$$

We note that for the fermion field, light-front propagator differs from the Feynmam propagator by an instantaneous propagator.

The free 1-fermion Green's function is given by

$$G(k^-) = \frac{\Lambda_+(k)}{\left(k^- - k^-_{on}+\frac{i\varepsilon}{k^+}\right)}, \tag{21}$$

where $\Lambda_\pm(k) = \frac{\pm\not{k}_{on}+m}{2m}\theta(\pm k^+)$.

V. COUPLED EQUATIONS FOR THE GREEN'S FUNCTIONS

Using the technique of the hierarchical equations [8] together with the paper of the reference [9], we built in the light-front Green's function for the two fermions system obtained from the solution of the covariant BS equation, that contains all two-body irreducible diagrams, with the exception of those including closed loops of bosons Ψ_1 and Ψ_2 and part of the cross-ladder diagrams, is given by:

$$G^{(2)}(K^-) = G_0^{(2)}(K^-) + G_0^{(2)}(K^-)VG^{(3)}(K^-)VG^{(2)}(K^-)\ , \tag{22}$$

$$G^{(3)}(K^-) = G_0^{(3)}(K^-) + G_0^{(3)}(K^-)VG_0^{(4)}(K^-)VG^{(3)}(K^-). \tag{23}$$

In the Yukawa model for fermions, the interaction operator acting between Fock-states differing by zero, one and two σ's, has matrix elements given by

$$\langle (q,s')k_\sigma|V|(k,s)\rangle = -2m(2\pi)^3\delta(q+k_\sigma-k)\frac{g_S}{\sqrt{q^+k^+_\sigma k^+}}\theta(k^+_\sigma)\overline{u}(q,s')u(k,s)\ , \tag{24}$$

$$\langle (q,s')k'_\sigma|V|(k,s)k_\sigma\rangle = -2(2\pi)^3\delta(q+k'_\sigma-k-k_\sigma)\delta_{s's}\frac{g_S^2}{\sqrt{k'^+_\sigma k^+_\sigma}}\frac{\theta(k'^+_\sigma)\theta(k^+_\sigma)}{k^+ + k^+_\sigma}\ , \tag{25}$$

$$\langle (q,s')k'_\sigma k_\sigma|V|(k,s)\rangle = -2(2\pi)^3\delta(q+k'_\sigma+k_\sigma-k)\delta_{s's}\frac{g_S^2}{\sqrt{k'^+_\sigma k^+_\sigma}}\frac{\theta(k'^+_\sigma)\theta(k^+_\sigma)}{k^+ - k^+_\sigma}\ . \tag{26}$$

The instantaneous terms in the two-fermion propagator give origin to Eqs. (25) and (26).

A systematic expansion by the consistent truncation of the light-front Fock space up to N particles in the intermediate states (boson 1, boson 2 and $N-2$ σ's) in the set of Eqs.(23) and (22), amounts to substitution $G^{(3)}(K^-) \cong G_0^{(3)}(K^-)$. The kernel of Eq.(22) still contains an infinite sum of light-front diagrams, that are obtained solving by Eq.(23). To obtain the ladder aproximation up to order g^2, Eq.(22), only the free and first order terms are kept in Eq.(23), with the restriction of only one and one boson covariant exchanges.Therefore, we have for Eq.(22)

$$G_{g^2}^{(2)}(K^-) = G_0^{(2)}(K^-) + G_0^{(2)}(K^-)VG_0^{(3)}(K^-)VG_{g^2}^{(2)}(K^-), \tag{27}$$

Taking the two-boson system as an example and restricting the intermediate state propagation up to 3-particles, we find that

$$G_{g^2}^{(2)}(K^-) = G_0^{(2)}(K^-) + G_0^{(2)}(K^-)VG_0^{(3)}(K^-)V\left\{G_0^{(2)}(K^-) + G_0^{(2)}(K^-)VG_0^{(3)}(K^-)VG_{g^2}^{(2)}(K^-)\right\} \tag{28}$$

The correction in order g^2 is given for $\Delta G_{g^2}^{(2)}(K^-) = G_0^{(2)}(K^-)VG_0^{(3)}(K^-)VG_0^{(2)}(K^-)$.

VI. BETHE-SALPETER EQUATION

We perform the quasi-potential reduction of two-body BSE's and present the coupled set of equations for the light-front Green's functions for gauge boson and fermion models, with the interaction Lagrangian respectively given by (13).

The bound state the Green's function has a pole $\lim_{K^- \to K^-_B} G^{(2)}(K^-) = \frac{|\psi_B\rangle\langle\psi_B|}{K^- - K^-_B}$, where $|\psi_B\rangle$ it is the wave-function of the bound state. Therefore, the homogeneous equation for the light-front two-body bound state wave-function is obtained the solution of

$$|\Psi_B >= G_0^{(2)}(K_B^-)VG^{(3)}(K_B^-)V|\Psi_B >, \tag{29}$$

the vertex function for the bound state wave-function is defined as

$$\Gamma_B(k_\perp, q^+) =< k,\ K-k| \left(G_0^{(2)}(K_B^-)\right)^{-1} |\Psi_B > . \tag{30}$$

The Green's function obtained from this equation, up to order g^2, reproduces the covariant two-body propagator between two light-front hypersurfaces. In this approximation, the vertex function satisfies the following integral equation,

$$\Gamma_B(\overrightarrow{q}_\perp, y) = \int \frac{dxd^2k_\perp}{x(1-x)} \frac{K^{(3)}(\overrightarrow{q}_\perp, y; \overrightarrow{k}_\perp, x)}{M_B^2 - M_0^2} \Gamma_B(\overrightarrow{k}_\perp, x), \tag{31}$$

where the momentum fractions are $y = q^+/K^+$ and $x = k^+/K^+$, with $0 < y < 1$. Where $\overrightarrow{K}_\perp = 0$, and $M_0^2 = K^+K^-_{(2)on} - K_\perp^2 = \frac{k_\perp^2 + m^2}{x(1-x)}$. The part of the kernel which contains only the propagation of virtual three particle states foward in the light-front time is obtained from Eq.(31) as,

$$\begin{aligned}\mathcal{K}^{(3)}(y, q_\perp; x, k_\perp) \ &= \ \frac{g^2}{16\pi^3} \frac{\Lambda_{+(1)}(q)\Lambda_{+(1)}(k)\Lambda_{+(2)}(K-q)\Lambda_{+(2)}(K-k)\theta(y-x)}{(y-x)^2\left(M_B^2 - \frac{\overrightarrow{q}_\perp^2 + m^2}{1-y} - \frac{\vec{k}_\perp^2 + m^2}{x} - \frac{(\overrightarrow{q}_\perp - \vec{k}_\perp)^2 + \mu^2}{y-x}\right)} + \\ &+ [k \leftrightarrow q] ,\end{aligned}$$

being $M_B^2 = K_B^+K_B^-$. We got the attention for the relationship between $|\Gamma_B\rangle$ and $\Gamma_B(\overrightarrow{q}_\perp, y)$, defined for $\Gamma_B(\overrightarrow{q}_\perp, y) = \sqrt{q^+(K^+ - q^+)}\,\langle\overrightarrow{q}_\perp, q^+\ |\Gamma_B\rangle$.

VII. CONCLUSION

Bethe-Salpeter equation (31) presents divergence in his kernel. We hoped to solve the problem of singularity that appears in the equation of Bethe-Salpeter with the inclusion of the singularity-softening prescription. Like this, to obtain finite Bethe-Salpeter. We are still accomplishing calculations for that aim at.

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[1] J.H.O.Sales, Tobias Frederico and B.M.Pimentel, Hadron Physiscs, 211-214 (2002).
[2] A.T. Suzuki and J.H.O.Sales, Mod. Phys. Lett. **A19** 1925 (2004).
[3] C.Itzykson and J.-B.Zuber, "Quantum field theory ", McGraw-Hill, 1980.
[4] A.T. Suzuki and J.H.O.Sales, Nucl.Phys. **A725** (2003) 139.
[5] B.M.Pimentel and A.T.Suzuki, Phys.Rev.**D42** , 2115 (1990).
[6] A.T. Suzuki and J.H.O. Sales, Mod. Phys. Lett. **A19** 2831 (2004).
[7] S.Mandelstam, Nucl.Phys. **B213**, 149 (1983). G. Leibbrandt, Phys. Rev. **D29**, 1699 (1984).
[8] T. Frederico, J.H.O. Sales, B.V. Carlson e P.U. Sauer, Nucl.Phys. **A737** (2004) 260.
[9] J.H.O.Sales, T. Frederico, B.V. Carlson e P.U. Sauer, Phys. Rev. **C 61** (2000) 044003.